Class X Session 2025-26
Subject - Mathematics (Basic)
Sample Question Paper - 04

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

Read the following instructions carefully and follow them:

1. This question paper contains 38 questions.
2. This Question Paper is divided into 5 Sections A, B, C, D and E.
3. In Section A, Questions no. 1-18 are multiple choice questions (MCQs) and questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21-25 are very short answer (VSA) type questions, carrying 02 marks each.
5. In Section C, Questions no. 26-31 are short answer (SA) type questions, carrying 03 marks each.
6. In Section D, Questions no. 32-35 are long answer (LA) type questions, carrying 05 marks each.
7. In Section E, Questions no. 36-38 are case study-based questions carrying 4 marks each with sub-parts of the values of 1,1 and 2 marks each respectively.
8. All Questions are compulsory. However, an internal choice in 2 Questions of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E.
9. Draw neat and clean figures wherever required.
10. Take $\pi = 22/7$ wherever required if not stated.
11. Use of calculators is not allowed.

Section A

1. The HCF of the smallest 2-digit number and the smallest composite number is **[1]**

   a) 4 b) 2

   c) 10 d) 20

2. What is the largest number that divides each one of 1152 and 1664 exactly? **[1]**

   a) 64 b) 128

   c) 32 d) 256

3. The roots of the quadratic equation $x^2 + 3x - 10 = 0$ are: **[1]**

   a) -5, 2 b) 5, 2

   c) 5, -2 d) -5, -2

4. If (-3, 2) is a solution of the linear equation 5x + 3 ky = 3, then the value of k is ________. **[1]**

a) 6

b) 3

c) 5

d) 2

5. The discriminant of the equation (2a + b) x = x^2 + 2ab is ________ [1]

a) $(2a + b)^2$

b) $(2a - b)^2$

c) $(2a + b^2)$

d) $(2a - b^2)$

6. If C(1, -1) is the mid-point of the line segment AB joining points A(4, x) and B(-2, 4), then value of x is: [1]

a) 6

b) -5

c) -6

d) 5

7. A vertical stick 1.8 m long casts a shadow 45 cm long on the ground. At the same time, what is the length of the shadow of a pole 6 m high? [1]

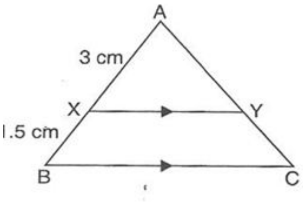
a) 2.4 m

b) 1.35 m

c) 1.5 m

d) 13.5 m

8. In the given figure $XY || BC$. If AX = 3cm, XB = 1.5 cm and BC = 6cm, then XY is equal to [1]



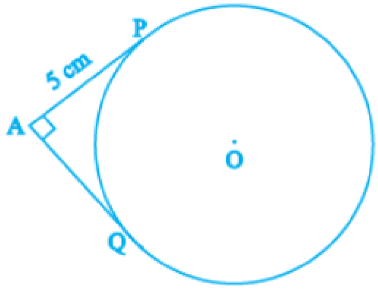
a) 6 cm.

b) 4.5 cm

c) 3 cm.

d) 4 cm.

9. The pair of tangents AP and AQ drawn from an external point to a circle with centre O are prependicular to each other and length of each tangent is 5 cm. The radius of the circle is [1]



a) 10 cm

b) 5 cm

c) 7.5 cm

d) 2.5 cm

10. If 5 tan A = 3, then the value of cot A is: [1]

a) $\frac{4}{5}$

b) $\frac{5}{3}$

c) $\frac{3}{4}$

d) $\frac{3}{5}$

11. The string of a kite in air is 50 m long and it makes an angle of 60^o with the horizontal. Assuming the string to be straight, the height of the kite from the ground is: [1]

a) $50\sqrt{3}$ m

b) $25\sqrt{3}$ m

c) $\frac{100}{\sqrt{3}}$ m

d) $\frac{50}{\sqrt{3}}$ m

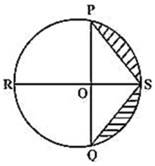
12. If $\frac{x}{3} = 2 \sin A$, $\frac{y}{3} = 2 \cos A$, then the value of $x^2 + y^2$ is: **[1]**

a) 6
b) 9
c) 36
d) 18

13. The hour hand of a clock is 6 cm long. The area swept by it between 11.20 am and 11.55 am is **[1]**

a) 11 cm^2
b) 2.75 cm^2
c) 10 cm^2
d) 5.5 cm^2

14. In the given figure PQ and RS are the perpendicular diameters of the circle whose centre is O and radius = 14 cm. the area of the shaded region is **[1]**



a) 28 cm^2
b) 35 cm^2
c) 60 cm^2
d) 112 cm^2

15. A bag contains cards numbered from 1 to 25. A card is drawn at random from the bag. The probability that the number on this card is divisible by both 2 and 3 is **[1]**

a) $\frac{3}{25}$
b) $\frac{1}{5}$
c) $\frac{4}{25}$
d) $\frac{2}{25}$

16. The median of first 8 prime numbers is **[1]**

a) 11
b) 13
c) 9
d) 7

17. The radii of the base of a cylinder and a cone are in the ratio 3 :4. If they have their heights in the ratio 2 : 3, the ratio between their volumes is **[1]**

a) 8 : 9
b) 3 : 4
c) 9 : 8
d) 4 : 3

18. If the difference of mode and median of a data is 24, then the difference of median and mean of the same data is: **[1]**

a) 34
b) 12
c) 24
d) 8

19. **Assertion (A):** Distance between (5, 12) and origin is 13 units. **[1]**

Reason (R): D = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** 3 is a rational number. **[1]**

Reason (R): The square roots of all positive integers are irrationals.

a) Both A and R are true and R is the correct
b) Both A and R are true but R is not the

explanation of A. correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

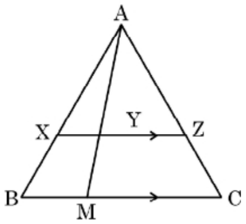
Section B

21. Is the pair of linear equation consistent/inconsistent? If consistent, obtain the solution graphically: x – y = 8; 3x – 3y = 16 **[2]**

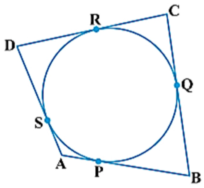
22. In $\triangle$ABC, D and E are the points on the sides AB and AC respectively such that DE||BC. If AD = 6x - 7, DB = 4x - 3, AE = 3x - 3 and EC = 2x - 1, find the value of x. **[2]**

OR

In the given figure, XZ is parallel to BC. AZ = 3 cm, ZC = 2 cm, BM = 3 cm and MC = 5 cm. Find the length of XY.



23. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC **[2]**

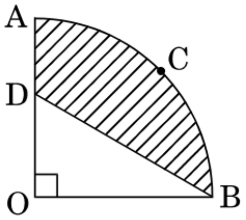


24. Prove that : $\frac{(\sin^4\theta+\cos^4\theta)}{1-2\sin^2\theta\cos^2\theta} = 1.$ **[2]**

25. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding: **[2]**
 i. minor segment
 ii. major sector.

OR

In Figure, OACB is a quadrant of a circle with centre O and radius 7 cm. If OD = 3 cm, then find the area of the shaded region.



Section C

26. Prove that $3+2\sqrt{5}$ is irrational. **[3]**

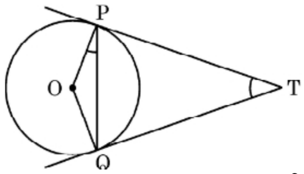
27. If the points P, Q(x, 7), R, S(6, y) in this order divide the line segment joining A(2, p) and B (7, 10) in 5 equal parts, find x, y and p. **[3]**

28. If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction? Solve the pair of the linear equation obtained by the elimination method. **[3]**

OR

Two years ago father was five times as old as his son. Two years later, his age will be 8 years more than three times the age of the son. Find the present ages of father and son.

29. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle$PTQ = 2 $\angle$OPQ. **[3]**



30. If $\tan\theta = \frac{12}{13}$, evaluate $\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$. **[3]**

OR

Prove that $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

31. Two different dice are thrown together. Find the probability that the numbers obtained **[3]**

i. have a sum less than 7

ii. have a product less than 16

iii. is a doublet of odd numbers.

Section D

32. A train travels at a certain average speed for a distance of 360 km. It would have taken 48 minutes less to travel the same distance if its speed was 5 km/hour more. Find the original speed of the train. **[5]**

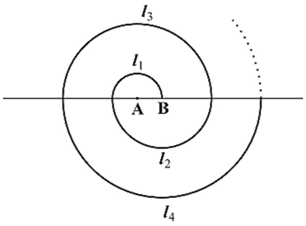
OR

Find the value of **k** for which the quadratic equation $(k + 1)x^2 - 2(3k + 1)x + (8k + 1) = 0$ has real and equal roots.

33. From the top of a vertical tower, the angles of depression of two cars in the same straight line with the base of the tower, at an instant are found to be 45° and 60°. If the cars are 100 m apart and are on the same side of the tower, find the height of the tower. **[5]**

34. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ... as shown in Figure. What is the total length of such a spiral made up of thirteen consecutive semicircles? $(Take\ \pi = \frac{22}{7})$ **[5]**

[**Hint:** Length of successive semicircles is l_1, l_2, l_3, l_4, ... with centres at A, B, A, B, ... respectively.]



OR

The ratio of the 11^{th} term to 17^{th} term of an A.P. is 3: 4. Find the ratio of 5^{th} term to 21^{st} term of the same A.P. Also, find the ratio of the sum of first 5 terms to that of first 21 terms.

35. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes. **[5]**

Number of mangoes	50-52	53-55	56-58	59-61	62-64
Number of boxes	15	110	135	115	25

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

A wooden toy is shown in the picture. This is a cuboidal wooden block of dimensions 14 cm $\times$ 17 cm $\times$ 4 cm. On its top there are seven cylindrical hollows for bees to fit in. Each cylindrical hollow is of height 3 cm and radius 2 cm.

i. Find the volume of wood carved out to make one cylindrical hollow. (1)

ii. Find the lateral surface area of the cuboid to paint it with green colour. (1)

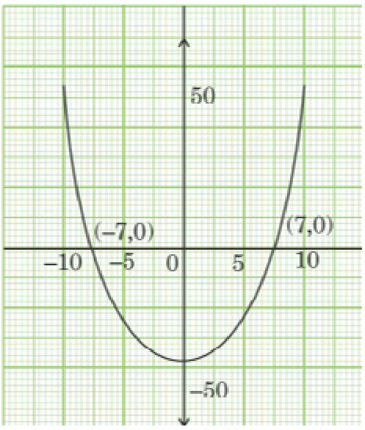
iii. Find the volume of wood in the remaining cuboid after carving out seven cylindrical hollows. (2)

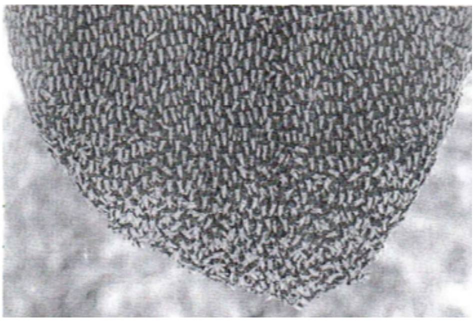
OR

Find the surface area of the top surface of the cuboid to be painted yellow. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

While playing in a garden, Samaira saw a honeycomb and asked her mother what is that. Her mother replied that it's a honeycomb made by honey bees to store honey. Also, she told her that the shape of the honeycomb formed is a mathematical structure. The mathematical representation of the honeycomb is shown in the graph.





i. How many zeroes are there for the polynomial represented by the graph given? (1)

ii. Write the zeroes of the polynomial. (1)

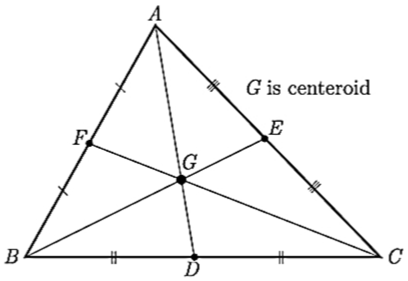
iii. If the zeroes of a polynomial $x^2 + (a + 1) x + b$ are 2 and -3, then determine the values of a and b. (2)

OR

If the square of difference of the zeroes of the polynomial $x^2 + px + 45$ is 144, then find the value of p. (2)

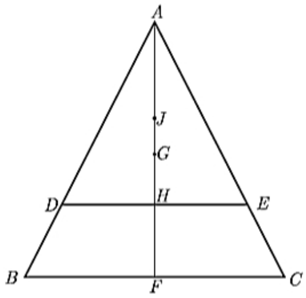
38. **Read the following text carefully and answer the questions that follow:** **[4]**

The centroid is the centre point of the object. It is also defined as the point of intersection of all the three medians. The median is a line that joins the midpoint of a side and the opposite vertex of the triangle. The centroid of the triangle separates the median in the ratio of 2 : 1. It can be found by taking the average of x-coordinate points and y-coordinate points of all the vertices of the triangle. See the figure given below



Here D, E and F are mid points of sides BC, AC and AB in same order. G is centroid, the centroid divides the median in the ratio 2 : 1 with the larger part towards the vertex. Thus AG : GD = 2 : 1

On the basis of above information read the question below. If G is Centroid of △ABC with height h and J is Centroid of △ADE. Line DE parallel to BC, cuts the △ABC at a height $\frac{h}{4}$ from BC. HF = $\frac{h}{4}$



i. What is the length of AH? (1)
ii. What is the distance of point A from point G? (1)
iii. What is the distance of point A from point J? (2)

OR

What is the distance GJ? (2)

Solution

Section A

1.

(b) 2

Explanation:

Smallest two digit number is 10 and smallest composite number is 4 .

Clearly, 2 is the greatest factor of 4 and 10, so their H.C.F. is 2.

2.

(b) 128

Explanation:

Largest number that divides each one of 1152 and 1664 = HCF (1152, 1664)

We know, 1152 = $2^7 \times 3^2$

1164 = $2^7 \times 13$

$\therefore$ HCF = 2^7 = 128

3. **(a)** -5, 2

Explanation:

p(x) = $x^2 + 3x - 10$

= $x^2 + 5x - 2x - 10$

= x(x + 5) - 2(x + 5)

= (x - 2) (x + 5)

p(x) = 0

x - 2 = 0 or x + 5 = 0

x = 2 or x = -5

4.

(b) 3

Explanation:

Since, (-3,2) is the solution of 5x + 3/cy = 3. So (-3, 2) satisfies it.

$\therefore$ 5 x (-3) + 3

$\Rightarrow -15 + 6k = 3 \Rightarrow k = \frac{18}{6} = 3$

5.

(b) $(2a - b)^2$

Explanation:

$(2a + b)x = x^2 + 2ab$

$x^2 - (2a + b)x + 2ab = 0$

$D = b^2 - 4ac$

$D = [-(2a + b)]^2 - 4 \times 1 \times 2ab$

$D = 4a^2 + b^2 + 4ab - 8ab$

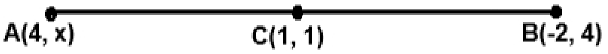
$D = 4a^2 + b^2 - 4ab$

$D = (2a - b)^2$

6.

(c) -6

Explanation:



Coordinate of C

$C\left(\frac{4-2}{2}, \frac{x+4}{2}\right)$

On comparing y coordinates.

$-1 = \frac{x+4}{2}$

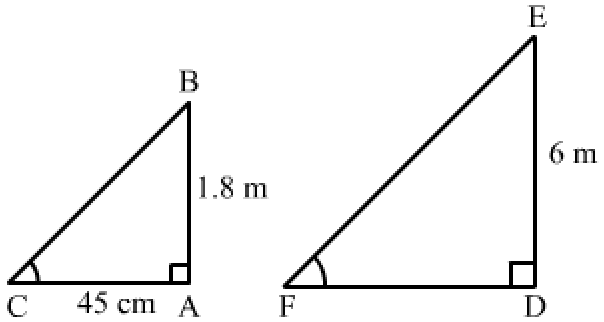
-2 = x + 4

x = -6

7.

(c) 1.5 m

Explanation:



Let AB and AC be the vertical stick and its shadow, respectively.

According to the question:

AB = 1.8 m

AC = 45 cm = 0.45 m

Again, let DE and DF be the pole and its shadow, respectively.

According to the question:

DE = 6 m

DF = ?

Now, in right-angled triangles ABC and DEF, we have:

∠BAC = ∠EDF = 90°

∠ACB = ∠DFE (Angular elevation of the Sun at the same time)

Therefore, by AA similarity theorem,

we get:△ABC∼ △DEF

$\Rightarrow \frac{AB}{AC} = \frac{DE}{DF} \Rightarrow \frac{1.8}{0.45} = \frac{6}{DF} \quad \Rightarrow DF = \frac{6\times0.45}{1.8} = 1.5\text{m}$

8.

(d) 4 cm.

Explanation:

Since $XY||BC$, then using Thales theorem,

$\Rightarrow \frac{\text{AX}}{\text{AB}} = \frac{\text{XY}}{\text{BC}}$

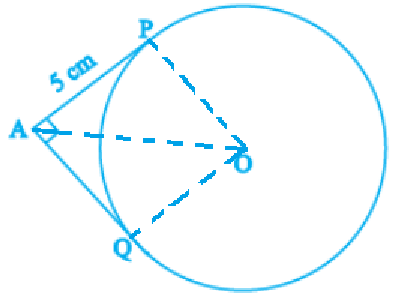
$\Rightarrow \frac{3}{4.5} = \frac{\text{XY}}{6}$

⇒XY = 4 cm

9.

(b) 5 cm

Explanation:



AP = AQ = 5 cm

(tangent from external point are equal)

Radii makes right angle with tangent

$\triangle \mathrm{APO} \cong \triangle \mathrm{AQO}$ (by R.H.S.)

As $\angle \mathrm{PAQ} = 90^\circ$, $So\ \angle \mathrm{PAO} = 45^\circ$

In $\triangle \mathrm{APO}$

$\tan 45^\circ = \frac{\mathrm{OP}}{\mathrm{AP}} = \frac{\mathrm{OP}}{5}$

$\Rightarrow \mathrm{OP} = 5\mathrm{cm}$

Hence, the radii of circle = 5 cm

10.

(b) $\frac{5}{3}$

Explanation:

5 tan A = 3

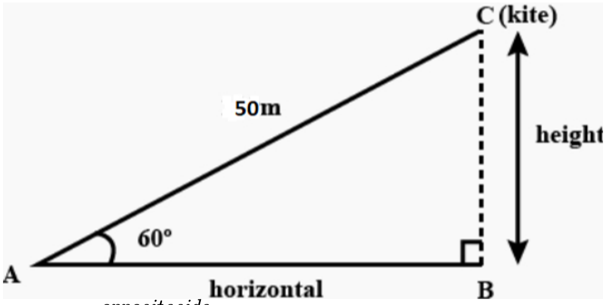
tan A = $\frac{3}{5}$

$\cot A = \frac{1}{\tan A} = \frac{5}{3}$

11.

(b) $25\sqrt{3}$ m

Explanation:



Sin(θ) = $\frac{opposite side}{hypotenuse}$

son60^{o} = $\frac{\mathrm{BC}}{\mathrm{AC}}$ = $\frac{h}{50}$

$\frac{\sqrt{3}}{2}$ = $\frac{h}{50}$ ($\because$ sin60^{o} = $\frac{\sqrt{3}}{2}$)

h = $25\sqrt{3}$ m

12.

(c) 36

Explanation:

$\frac{x}{3} = 2\sin A, \frac{y}{3} = 2\cos A$

$x = 6\sin A, y = 6\cos A$

$x^2 + y^2 = (6\sin A)^2 + (6\cos A)^2$

$= 36\sin^2 A + 36\cos^2 A$

$= 36\left(\sin^2 A + \cos^2 A\right)$

= 36(1)

= 36

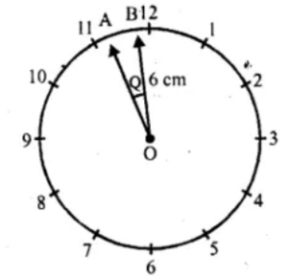
13.

(d) 5.5 cm^2

Explanation:

Length of hour hand of a clock (r) = 6 cm

Time 11.20 am to 11.55 am = 35 minute = $\frac{35}{60}$h



$\therefore$ In 1 hour the hour hand rotates 30°.

Thus, central angle of the sector = $30 \times \frac{35}{60} = 17.5^\circ$

$\therefore$ Area of the sector swept by the hour hand = $\frac{17.5}{360} \times \frac{22}{7} \times 6 \times 6$ cm^2

$= \frac{2.5\times22}{10}\text{cm}^2 = 5.5\text{cm}^2$

14.

(d) 112 cm^2

Explanation:

For Triangle POS,

PO = OS = 14cm

Now Get PS by pythagoras theorem.

Again, Required area = $\frac{1}{4}$Area of Circle - Area POS

15.

(c) $\frac{4}{25}$

Explanation:

Total number of outcomes = 25

The number which is divisible by both 2 and 3 are 6, 12, 18, 24

Number of favourable outcomes = 4

Probability of number which is divisible by both 2 and 3 = $\frac{4}{25}$

16.

(c) 9

Explanation:

First 8 prime numbers are follows:

2, 3, 5, 7, 11, 13, 17, 19

N = 8 (even)

$\therefore$ Median $= \frac{\left(\frac{8}{2}\right)^{\text{th}} \text{ value} + \left(\frac{8}{2}+1\right)^{\text{th}} \text{ value}}{2}$

$= \frac{4^{\text{th}} \text{ value} + 5^{\text{th}} \text{ value}}{2}$

$= \frac{7+11}{2}$

$= \frac{18}{2}$

= 9

17.

(c) 9 : 8

Explanation:

Let the radii of the base of the cylinder and cone be 3r and 4r and their heights be 2h and 3h, respectively.

Then, ratio of their volumes = $\frac{\pi(3r)^2\times(2h)}{\frac{1}{3}\pi(4r)^2\times(3h)}$

= $\frac{9r^2\times2\times3}{16r^2\times3}$

= $\frac{9}{8}$

= 9 : 8

18.

(b) 12

Explanation:

Given,

mode - median = 24

median - mean = ?

we know that,

mode = 3 median - 2 mean

mode = median + 2 median - 2 mean

mode - median = 2 median - 2 mean

24 = 2 (median - mean)

median - mean = $\frac{24}{2}$ = 12

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Distance of point (5, 12) from 8 origin is given, d = $\sqrt{(5-0)^2+(12-0)^2}$

= $\sqrt{25+144}=\sqrt{169}$ = 13

20.

(c) A is true but R is false.

Explanation:

Here, reason is not true.

$\sqrt{9}$ = ±3, which is not an irrational number.

A is true but R is false.

Section B

21. x - y = 8.................(1)

3 x - 3 y = 16.............(2)

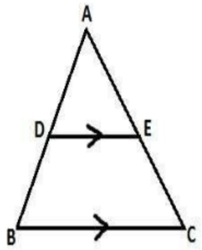
Here, $a_1=1, b_1=-1, c_1=-8$

$a_2=3, b_2=-3, c_2=-16$

We see that $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

Hence, the lines represented by the equations(1) and (2) are parallel.

Therefore, equations (1) and (2) have no solution, i.e., the given pair of linear equation is inconsistent.

22. 

Given: In $\triangle$ABC, DE || BC. Also AD = 6x - 7, DB = 4x - 3, AE = 3x - 3 and EC = 2x - 1

By basic proportionality theorem,

$\frac{AD}{DB}=\frac{AE}{EC}$

$\Rightarrow\frac{6x-7}{4x-3}=\frac{3x-3}{2x-1}$

$\Rightarrow$ (6x - 7)(2x - 1) = (3x - 3)(4x - 3)

$\Rightarrow$ $12x^2$ - 6x - 14x + 7 = $12x^2$ - 9x - 12x + 9

$\Rightarrow$ -20x + 7 = -21x + 9

$\Rightarrow$ -20x + 21x = 9 - 7

$\Rightarrow$ x = 2

OR

Given that,

In the figure the triangle ABC

XZ||BC and the length of AZ = 3 cm, ZC = 2 cm, BM = 3 cm and MC = 5 cm.

From $\triangle$ABC and $\triangle$AXZ

$\angle$AXZ = $\angle$ABC [by corresponding angles]

$\angle$AZX = $\angle$ACB [by corresponding angles]

By basic proportionality theorem $\triangle$ABC and $\triangle$AXZ are similar.

So,

$\frac{YZ}{MC} = \frac{AZ}{ZC}$

$\frac{YZ}{5} = \frac{3}{2}$

YZ = $\frac{15}{2}$

Then,

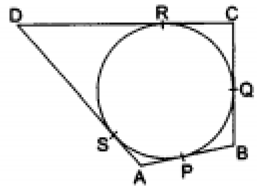
$\frac{XZ}{BC} = \frac{AZ}{ZC}$

$\frac{XY+YZ}{BM+MC} = \frac{AZ}{ZC}$

$\frac{XY+\frac{15}{2}}{3+5} = \frac{3}{2}$

XY + $\frac{15}{2} = \frac{24}{2}$

XY = $\frac{9}{2}$ = 4.5 cm

23. 

We know that the lengths of tangents drawn from an exterior point to a circle are equal.

AP = AS, ... (i) [tangents from A]

BP = BQ, ... (ii) [tangents from B]

CR = CQ, ... (iii) [tangents from C]

DR = DS. ... (iv) [tangents from D]

AB + CD = (AP + BP) + (CR + DR)

= (AS + BQ) + (CQ + DS) [using (i), (ii), (iii), (iv)]

= (AS + DS) + (BQ + CQ)

= AD + BC.

Hence, AB + CD = AD + BC.

24. We have,

$\frac{(\sin^4\theta+\cos^4\theta)}{1-2\sin^2\theta\cos^2\theta}$

$= \frac{(\sin^2\theta)^2+(\cos^2\theta)^2}{1-2\sin^2\theta\cos^2\theta}$

$= \frac{(\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta\cos^2\theta}{1-2\sin^2\theta\cos^2\theta}\left[(a+b)^2 = a^2+b^2+2ab\right]$

$= \frac{1-2\sin^2\theta\cos^2\theta}{1-2\sin^2\theta\cos^2\theta}\left[sin^2\theta+cos^2\theta = 1\right]$

$= 1$ Hence proved

25. i. r = 10 cm, $\theta = 90^\circ$

Area of minor sector = $\frac{\theta}{360} \times \pi r^2$

$= \frac{90}{360} \times 3.14 \times 10 \times 10 = 78.5\text{cm}^2$

Area of $\triangle OAB = \frac{OA \times OB}{2}$

$= \frac{10\times10}{2} = 50\text{cm}^2$

$\therefore$ Area of the minor segment

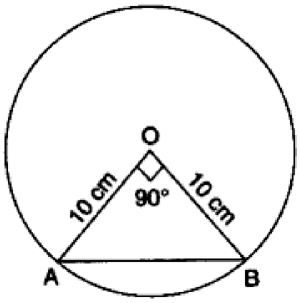
= Area of minor sector - Area of $\triangle OAB$

$= 78.5\text{cm}^2 - 50\text{cm}^2 = 28.5\text{cm}^2$

ii. Area of major sector = $\pi r^2 - area\ of\ minor\ sector$

$= 3.14 \times 10 \times 10 - 78.5$

$= 314 - 28.5 = 285.5\text{cm}^2$



OR

Area of quadrant = $\frac{1}{4}\pi(7)^2 = \frac{49}{4}\pi$ cm^2

Area of triangle = $\frac{1}{2} \times 7 \times 3$ = $\frac{21}{2}$ cm^2

Area of shaded region = $\frac{49}{4}\pi - \frac{21}{2}$

= $\frac{7}{2}\left(\frac{7}{2}\pi - 3\right)$ cm^2 or 28 cm^2

Section C

26. Let us assume, to the contrary, that is $3 + 2\sqrt{5}$ rational.

That is, we can find coprime integers a and b $(b \neq 0)$ such that

$3 + 2\sqrt{5} = \frac{a}{b}$ Therefore, $\frac{a}{b} - 3 = 2\sqrt{5}$

$\Rightarrow \frac{a-3b}{b} = 2\sqrt{5}$

$\Rightarrow \frac{a-3b}{2b} = \sqrt{5} \Rightarrow \frac{a}{2b} - \frac{3}{2}$

Since a and b are integers,

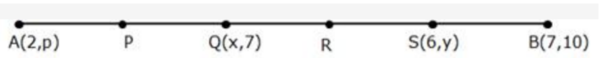
We get $\frac{a}{2b} - \frac{3}{2}$ is rational, also so $\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

This contradiction arose because of our incorrect

assumption that $3 + 2\sqrt{5}$ is rational.

So, we conclude that $3 + 2\sqrt{5}$ is irrational.

27. 

Consider QB.

The point S divides QB in the ratio 2:1

$6 = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$

$\Rightarrow 6 = \frac{2(7) + 1(x)}{2+1}$

$\Rightarrow 6 = \frac{x+14}{3}$

$\Rightarrow$18 = x + 14

$\Rightarrow$ x = 4

$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$

$\Rightarrow y = \frac{2(10) + 1(7)}{2+1}$

$\Rightarrow y = \frac{20+7}{3}$

$\Rightarrow$ y = 9

Consider AS.

The point Q divides AS in the ratio 1:1.

$7 = \frac{y_1 + y_2}{2}$

$\Rightarrow 7 = \frac{p+9}{2}$

$\Rightarrow$14 = p+ 9

$\Rightarrow$ p = 5

So, x = 4, y = 9, p = 5.

28. Let the fraction be $\frac{x}{y}$

Then, according to the question,

$\frac{x+1}{y-1}=1$ ......(1)

$\frac{x}{y+1}=\frac{1}{2}$ ....(2)

⇒x + 1 = y - 1 ....(3)

2x = y + 1................(4)

⇒x - y = - 2................(5)

2x - y = 1......................(^)

Substituting equation (5) from equation (6), we get x =3

Substituting this value of x in equation (5), we get

3 - y = -2

⇒ y = 3 + 2

⇒ y = 5

Hence, the required fraction is $\frac{3}{5}$

Verification: Substituting the value of x = 3 and y = 5,

we find that both the equations(1) and (2) are satisfied as shown below:

$\frac{x+1}{y-1}=\frac{3+1}{5-1}=\frac{4}{4}=1$

$\frac{x}{y+1}=\frac{3}{5+1}=\frac{3}{6}=\frac{1}{2}$

Hence, the solution is correct.

OR

Let the age of father is x years and that of son is y years.

Then by the given question,

Two years ago father was five times as old as his son

$x-2=5(y-2)$

or, $x-5y=-10+2$

or, $x-5y=-8$

or, $x=5y-8$

Two years later, his age will be 8 years more than three times the age of the son

$x+2=3(y+2)+8$

or, $x-3y=6+8-2$

or, $5y-8-3y=12$

or, $2y=12+8$

or, $y=\frac{20}{2}$

or, $y=10$

then x=5y-8=50-8=42

Then, the age of father is 42 yrs. and the age of son is 10 yrs.

29. Given : A circle with centre O and an external point T and two tangents TP and TQ to the circle, where P, Q are the points of contact.

To Prove: ∠PTQ = 2∠OPQ

Proof: Let ∠PTQ = θ

Since TP, TQ are tangents drawn from point T to the circle.

TP = TQ

∴ TPQ is an isoscles triangle

∴ ∠TPQ = ∠TQP = $\frac{1}{2}$ (180^o - θ) = 90^o - $\frac{\theta}{2}$

Since, TP is a tangent to the circle at point of contact P

∴ ∠OPT = 90^o

∴ ∠OPQ = ∠OPT - ∠TPQ = 90^o - (90^o - $\frac{1}{2}$ θ) = $\frac{\theta}{2}$ = $\frac{1}{2}$∠PTQ

Thus, ∠PTQ = 2∠OPQ

30. We have, $\tan\theta=\frac{12}{13}$

Now, $\frac{2\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}=\frac{\frac{2\sin\theta\cos\theta}{\cos^2\theta}}{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}$ [dividing numerator and denominator by $\cos^2\theta$]

$= \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\times\frac{12}{13}}{1-\left(\frac{12}{13}\right)^2} = \frac{\frac{24}{13}}{1-\frac{144}{169}} = \frac{\frac{24}{13}}{\frac{25}{169}} = \frac{24}{13} \times \frac{169}{25} = \frac{312}{25}$

Hence, $\frac{2\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta} = \frac{312}{25}$

OR

First, we will show that, $\frac{1+\tan^2 A}{1+\cot^2 A}$ = $\tan^2$ A

LHS = $\frac{1+\tan^2 A}{1+\cot^2 A}$ = $\frac{1+\tan^2 A}{1+\frac{1}{\tan^2 A}}$ = $\frac{1+\tan^2 A}{\frac{1+tan^2 A}{\tan^2 A}}$

= $(1+tan^2 A) \times \frac{tan^2 A}{1+tan^2 A}$

= $\tan^2$ A = RHS ....(i)

Now, we will show that, $\left(\frac{1-\tan A}{1-\cot A}\right)^2$ = $\tan^2$ A

LHS = $\left(\frac{1-\tan A}{1-\cot A}\right)^2$ = $\left(\frac{1-\tan A}{1-\frac{1}{tan A}}\right)^2$ = $\left(\frac{1-\tan A}{\frac{tan A-1}{tan A}}\right)^2$

= $\left[(1-\tan A)\times\left(\frac{\tan A}{-(1-\tan A)}\right)\right]^2$

= $(\tan A)^2$ = $\tan^2$ A = RHS ...(ii)

Hence, from (i) and (ii),

$\frac{1+\tan^2 A}{1+\cot^2 A}$ = $\left(\frac{1-\tan A}{1-\cot A}\right)^2$ = $\tan^2$ A

Hence proved.

31. When 2 dice are rolled,

The possible outcomes are :

(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
(4,1),(4,2),(4,3),(3,4),(3,5),(3,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)

$\therefore$Total number of outcomes = 36

i. Let A be the event of getting the numbers whose sum is less than 7.

Number of favourable outcomes =15

Favourable outcomes are (1,1),(1,2),(1,3),(1,4),(1,5),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(4,1),(4,2) and (5,1).

$\therefore P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{15}{36} = \frac{5}{12}$

ii. Let B be the event of getting the numbers whose product is less than 16.

Number of favourable outcomes= 25

Favourable outcomes are (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(3,1),(3,2),(3,3),(3,4),(3,5),(4,1),(4,2),(4,3),(5,1),(5,2),(5,3),(6,1) and (6,2).

$\therefore P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{25}{36}$

iii. Let C be the event of getting the numbers which are doublets of odd numbers.

Number of favourable outcomes=3

Favourable outcomes are (1,1),(3,3) and (5,5).

$\therefore P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{3}{36} = \frac{1}{12}$

Section D

32. Given that a train travelling at a uniform speed for 360 km

Let the original speed of the train be x km/hr

Time taken $= \frac{\text{Distance}}{\text{Speed}} = \frac{360}{x}$

Time taken at increased speed $= \frac{360}{x+5}$ hours.

According to the question

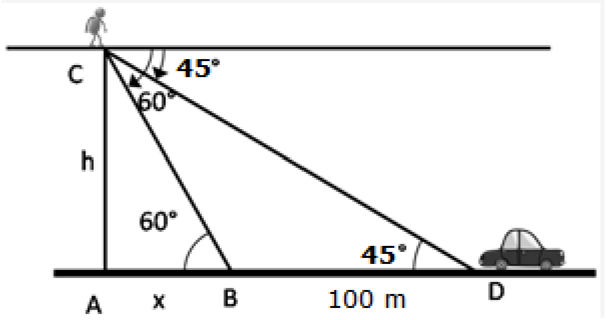
$\frac{360}{x} - \frac{360}{x+5} = \frac{48}{60}$

$360\left[\frac{1}{x} - \frac{1}{x+5}\right] = \frac{4}{5}$

$or, \frac{360(x+5-x)}{x^2+5x} = \frac{4}{5}$
$or, \frac{1800}{x^2+5x} = \frac{4}{5}$
$\Rightarrow \quad x^2 + 5x - 2250 = 0$
$\Rightarrow \quad x^2 + (50 - 45)x - 2250 = 0$
$\Rightarrow \quad x^2 + 50x - 45x - 2250 = 0$
$\Rightarrow \quad (x + 50)(x - 45) = 0$
Either x = - 50 or x = 45
As speed cannot be negative
$\therefore$ Original speed of train = 45 km/hr.

OR

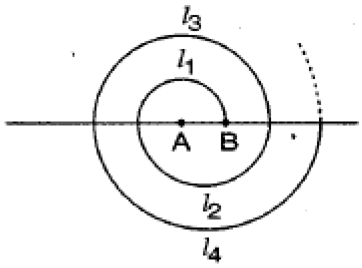
For real and equal roots.
$[-2(3k + 1)]^2 - 4(k + 1)(8k + 1) = 0$
$\Rightarrow k^2 - 3k = 0$
$\therefore k = 0, k = 3$

33. 

Let AC be the tower of height = h m.
In right $\triangle$DAC,
$\cot 45^\circ = \frac{AD}{AC}$
$\Rightarrow 1 = \frac{x+100}{h}$
$\Rightarrow x + 100 = h$
$\Rightarrow x = h - 100$ ...(i)
In right ΔBAC,
$\cot 60^\circ = \frac{AB}{AC}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{h}$
$\Rightarrow x = \frac{h}{\sqrt{3}}$ ......(ii),
From (i) and (ii),
$h - 100 = \frac{h}{\sqrt{3}}$
$\Rightarrow h - \frac{h}{\sqrt{3}} = 100$
$\Rightarrow h\left(1 - \frac{1}{\sqrt{3}}\right) = 100$
$\Rightarrow h\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) = 100$
$\Rightarrow h = \frac{100\sqrt{3}}{\sqrt{3}-1}$
$h = \frac{100\sqrt{3}}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$
$\Rightarrow h = \frac{300+100\sqrt{3}}{2}$
$= 100\left(\frac{3+\sqrt{3}}{2}\right)$
$= 50(3 + \sqrt{3})$
$= 50(3 + 1.73)$
$= 236.5m$
Hence, the height of the tower is 236.5 m.

34. According to question we are given that a spiral is made up of successive semi-circles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ...as shown in Fig.

Let $l_1, l_2, l_3, l_4,..l_{13}$ be the lengths (circumferences) of semi-circles of radii r_1= 0.5 cm, r_2 =1.0 cm, r_3 = 1.5 cm, r_4 = 2.0 cm, r_5 = 2.5 cm,... respectively.



Now, Semi-perimeter of circle = $\pi \cdot r$

Therefore,

$l_1 = \pi r_1 = \pi \times 0.5 = \frac{\pi}{2}\text{cm}$

$l_2 = \pi r_2 = \pi \times 1 = 2\left(\frac{\pi}{2}\right)\text{ cm}$

$l_3 = \pi r_3 = \pi \times \frac{3}{2} = 3\left(\frac{\pi}{2}\right)\text{ cm}$

$l_4 = \pi r_4 = \pi \times 2 = 4\left(\frac{\pi}{2}\right)\text{ cm}$

and

$l_{13} = \pi r_{13} = \pi \times \frac{13}{2}\text{cm} = 13\left(\frac{\pi}{2}\right)\text{ cm}$

Therefore total length of the spiral = $l_1 + l_2 + l_3 + ... + l_{13}$

$= \left\{\frac{\pi}{2} + 2\left(\frac{\pi}{2}\right) + 3\left(\frac{\pi}{2}\right) + \cdots + 13\left(\frac{\pi}{2}\right)\right\}$

$= \frac{\pi}{2}(1 + 2 + 3 + \cdots + 13)$

$= \frac{\pi}{2} \times \frac{13}{2}(1 + 13) \quad \left[\text{Using } S_n = \frac{n}{2}(a + l)\right]$

$= \frac{\pi}{2} \times \frac{13}{2} \times 14 = \frac{1}{2} \times \frac{22}{7} \times 13 \times 7 = 143\text{cm}$

which is required length of the spiral made up of thirteen consecutive semi-circles.

OR

Given $\frac{a+10d}{a+16d} = \frac{3}{4}$

$\Rightarrow$ 4a + 40d = 3a + 48d

$\Rightarrow$ a = 8d

therefore $\frac{a_5}{a_{21}} = \frac{a+4d}{a+20d} = \frac{3}{7}$ using (i)

$a_5 : a_{21} = 3 : 7$

$\frac{s_5}{s_{21}} = \frac{\frac{5}{2}(2a+4d)}{\frac{21}{2}(2a+20d)} = \frac{5\times 20d}{21\times 36d} = \frac{25}{189}$

Therefore, $S_5 : S_{21} = 25 : 189$

35. Since value of number of mangoes and number of boxes are large numerically. So we use step-deviation method

True Class Interval	**No. of boxes(f_i)**	**Class mark(x_i)**	$u_i = \frac{x_i - a}{h}$	**f_iu_i**
49.5-52.5	15	51	-2	-30
52.5-55.5	110	54	-1	-110
55.5-58.5	135	57	0	0
58.5-61.5	115	60	1	115
61.5-64.5	25	63	2	50
	$\sum f_i$ = 400			$\sum f_iu_i$ = 25

Let assumed mean (a) = 57,

h = 3 ,

$\therefore \bar{u} = \frac{\sum f_iu_i}{\sum f_i} = \frac{25}{400} = 0.0625$ (approx.)

Using formula, Mean $(\bar{x})$ = a + h$\bar{u}$

= 57 + 3 (0.0625

= 57 + 0.1875

= 57.1875

= 57.19 (approx)

Therefore, the mean number of mangoes is 57.19

Section E

36. i. Volume of wood carved out to make one hollow

$= \frac{22}{7} \times 2 \times 2 \times 3 = \frac{264}{7}$ cm^3 or 37.7 cm^3

ii. LSA of cuboid = 2(14 $\times$ 4 + 17 $\times$ 4) = 248 cm^2.

iii. Volume of 7 cylindrical hollows = 264 cm^3.

Volume of original cuboid = 14 $\times$ 17 $\times$ 4 = 952 cm^3.

$\therefore$ Volume of remaining solid = 952 – 264 = 688 cm^3.

OR

Area of top surface to be painted = (l $\times$ b) – 7 $\times$ πr^2

= (14 $\times$ 17) – ($\frac{22}{7}$ $\times$ 4 $\times$ 7)

= 150 cm^2

37. i. Two

ii. 7 and -7

iii. -(a + 1) = 2 + (-3) $\Rightarrow$ a = 0

b = 2 $\times$ (–3) $\Rightarrow$ b = -6

OR

Let α and β be the zeroes of given polynomial

Here, $\alpha + \beta$ = -p and $\alpha\beta$ = 45

$(\alpha - \beta)^2$ = 144

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta$ = 144

$\Rightarrow (-p)^2 - 4 \times 45$ = 144

$\Rightarrow$ p = $\pm$18

38. i. $\therefore$ AF = h (Given)

$\therefore$ AF = AH + HF

h = AH + $\frac{h}{4}$

AH = h - $\frac{h}{4}$

AH = $\frac{3h}{4}$

ii. $\because$ AF = h (Given)

$\therefore$ AG = $\frac{2}{3}$ AF

$\because$ centroid divide the median in 2 : 1

iii. AH = $\frac{3h}{4}$

J is centroid of $\triangle$ADE

AJ : JH = 2 : 1

let AJ = 2x and JH = x

2x + x = $\frac{3h}{4}$

x = $\frac{h}{4}$

AJ = 2 $\times$ $\frac{h}{4}$ = $\frac{h}{2}$

AG = AJ + GJ

$= \frac{h}{2} + \frac{h}{6}$

$= \frac{2h}{3}$

But AJ = $\frac{h}{2} \times \frac{2}{3}$

AJ = $\frac{3}{4}$ AG

OR

GJ = AG - AJ

= AG - $\frac{3}{4}$ AG

GJ = $\frac{1}{4}$ AG